

## 4.1

# Don't Take This Out of Context

## Analyzing Polynomial Functions

### LEARNING GOALS

In this lesson, you will:

- Analyze the key characteristics of polynomial functions in a problem situation.
- Determine the average rate of change of a polynomial function.
- Solve equations and inequalities graphically.

### KEY TERM

- average rate of change

The *kill screen* is a term for a stage in a video game where the game stops or acts oddly for no apparent reason. More common in classic video games, the cause may be a software bug, a mistake in the program, or an error in the game design. A well-known kill screen example occurs in the classic game *Donkey Kong*. When a skilled player reaches level 22, the game stops just seconds into Mario's quest to rescue the princess. Game over even though the player did not do anything to end the game!

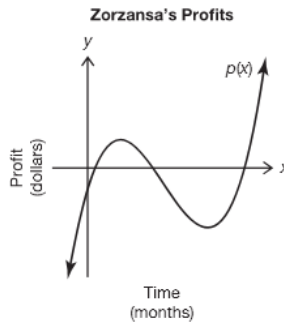
Video game technology has advanced dramatically over the last several decades, so these types of errors are no longer common. Games have evolved from simple movements of basic shapes to real-time adventures involving multiple players from all over the globe.

How do you think video games will change over the next decade?

**PROBLEM 1** Play Is Our Work



The polynomial function  $p(x)$  models the profits of Zorzansa, a video game company, from its original business plan through its first few years in business.



1. Label the portion(s) of the graph that model each of the memorable events in the company's history by writing the letter directly on the graph. Explain your reasoning.

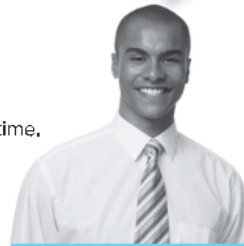
a. The Chief Executive Officer anxiously meets with her accountant.

b. The highly anticipated game, *Rage of Destructive Fury II*, is released.

c. The company opens its doors for business for the first time.

d. The company reaches its first short-term sales goal just as the holiday shopping season ends.

Several answers may be correct as long as you can defend your reasoning. The events are not necessarily written in chronological order.



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- e. The company breaks even.
- f. Members of the Board of Directors get in a heated debate over the next move the company should make.
- g. The game design team is fired after their 2 game releases, *Leisurely Sunday Drive* and *Peaceful Resolution*, delight many parents but sell poorly.

h. A large conglomerate buys the company.

No model is perfect for real data, though some are more appropriate than others. In what ways does this cubic model make sense? In what ways does it not make sense?

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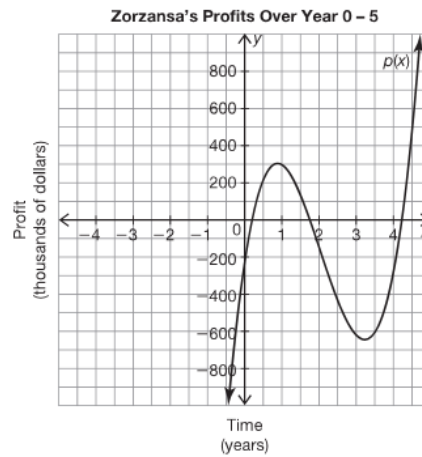


2. Do you think this cubic function is an appropriate model for this scenario? Explain your reasoning.



**PROBLEM 2** There's Nothing "Average" About This Rate of Change


The cubic function  $p(x)$  models Zorzansa's total profits over the first five years of business.



1. Use the graph to estimate when Zorzansa's achieved each profit. Then explain how you determined your estimate.

- a. \$800,000
- b. \$200,000
- c. greater than \$200,000
- d. the company is losing money
- e. the company is making a profit.

What is the maximum number of solutions for a given profit?





2. Avi and Ariella disagree about the end behavior of the function.

Avi

The end behavior is incorrect. As time increases, profit approaches infinity. It doesn't make sense that the profits are increasing before the company even opens.

Ariella

The end behavior is correct. The function is cubic with a positive  $a$ -value. This means as  $x$  approaches infinity,  $y$  approaches infinity. As  $x$  approaches negative infinity,  $y$  also approaches negative infinity.

Who is correct? Explain your reasoning.

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The **average rate of change** of a function is the ratio of the independent variable to the dependent variable over a specific interval. The formula for average rate of change is  $\frac{f(b) - f(a)}{b - a}$  for an interval  $(a, b)$ . The expression  $a - b$  represents the change in the input of the function  $f$ . The expression  $f(b) - f(a)$  represents the change in the function  $f$  as the input changes from  $a$  to  $b$ .

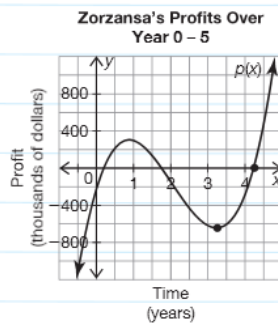
You've already calculated average rates of change when determining slope, miles per hour, or miles per gallon. It's the change in  $y$  divided by the change in  $x$ .



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You can determine the average rate of change of Zorzansa's profit for the time interval  $(3.25, 4.25)$ .



Substitute the input and output values into the average rate of change formula.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(4.25) - f(3.25)}{4.25 - 3.25}$$

Simplify the expression.

$$= \frac{0 - (-600)}{1}$$

$$= \frac{600}{1} = 600$$

The average rate of change for the time interval  $(3.25, 4.25)$  is approximately \$600,000 per year.

3. Analyze the worked example.
- Explain why the average rate of change is \$600,000 per year, and not \$600 per year.
  - Explain why the average rate of change is positive over this interval.
  - What does the average rate of change represent in this problem situation?



4. Determine the average rate of change of Zorzansa's profits for the time interval (1, 3).



5. Sam has a theory about the average rate of change.

 **Sam**

*I can quickly estimate the average rate of change for intervals that are above and below the x-axis because they add to zero. For example, at year 1, the profit is about \$225,000 and at year 2.5 the profit is about \$-225,000. Therefore, the average rate of change for the time interval (1, 2.25) is approximately \$0.*

Describe the error in Sam's reasoning.

6. After 4.5 years would you consider Zorzansa as a successful business?  
Explain your reasoning.

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Be prepared to share your solutions and methods.